

Faculty of Science

B.Sc (Mathematics) II-Year, CBCS -III Semester Backlog Examinations, January 2021

PAPER: REAL ANALYSIS

Time: 2 Hours

Max Marks: 80

I. Answer any **FOUR** of the following questions

(4x20=80 Marks)

1. (i) If $\{s_n\}$ converges to s and $\{t_n\}$ converges to t . then prove that $\{s_n+t_n\}$ converges to $s+t$.

(ii) Prove that Every Convergent sequence is bounded.

2. Show that the sequence s_n defined recursively by $s_1 = \sqrt{2}$, $s_{n+1} = \sqrt{2s_n}$ converges to 2.

3. Test the convergence of the series:

$$(i) \sum \frac{n^3}{3^n} \quad (ii) \sum_{n=2}^{\infty} \frac{\log n}{2n^3+1}$$

4. State and prove alternating series theorem.

5. Let $f_n: S \rightarrow R$ be a sequence of functions on a set $S \subseteq R$. Then show that $\{f_n\}$ is uniformly convergent iff the sequence $\{f_n\}$ is uniformly Cauchy on S .

6. Find the radius of convergence and determine the exact interval of convergence of the power series $\sum \left(\frac{3^n}{n 4^n}\right) x^n$

7. (i) Show that every monotonic function f on $[a, b]$ is integrable.

(ii) Show that if f is integrable on $[a, b]$ then $|f|$ is integrable on $[a, b]$

$$\text{and } \left| \int_a^b f \right| \leq \int_a^b |f|$$

8. Let $f(x) = x^2$ for all $x \in [0, b]$ then show that f is integrable on $[0, b]$

$$\text{and } \int_a^b f(x) dx = \frac{b^3}{3}$$
