Code No. SIIIMAT/E

Faculty of Science

B.Sc. (Mathematics) II Year, CBCS – III-Semester Examinations, Dec 2017

Paper-III (Real Analysis)

Time: 3 hours | Max Marks: 80

Section-A

I. Answer any **Five** questions. (5x4=20 Marks)

1. Prove that every convergent sequence is bounded.
2. Show that every monotone bounded sequence is convergent.
3. Define Alternating Series and State Alternating Series Theorem.
4. Test the convergence of $\Sigma\left(\frac{n}{n^2+3}\right)$.
5. Define sequence of functions and its convergence pointwise.
6. State and prove weistrass M-test.
7. $If\ f(x) = k\ \forall\ x \in [a,b] then\ prove\ that\ \int_a^b f(x)dx = k(b-a)$.
8. Prove that every monotonic function 'f' on $[a,b]$ *is integrable on* $[a,b]$.

Section-B

II. Answer all the questions. (4x15=60 Marks)

9. (a) State and Prove Squeeze Theorem.

(OR)

(b) Define a Monotonic Sequence. Find whether the sequence $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots\ldots\ldots + \frac{1}{n+n}, n \in N$. is monolonically increasing or decreasing. Is it convergent?

10. (a) State and prove cauchy's general principle of convergence.

(OR)

(b) Write the statement of Ratio test. Discuss the convergence of the series $\sum_{n=2}^{\infty}\left(\frac{\log n}{n}\right)$.

11. (a)Let $f_n: s \to R$, be a sequence of function on a set $S \subseteq R$, then (f_n) is uniformly convergent iff the sequence (f_n) is uniformly Cauchy on S.

(OR)

(b) Define and find the radius of convergence of the power series

i) $\sum_{n=1}^{\infty}\left(\frac{1}{n}x^n\right)$ and ii) $\sum_{n=0}^{\infty} n!\, x^n$.

12. (a) Prove that a bounded function 'f' on [a, b] is integrable iff for each $\epsilon > 0$, $\exists$ a partition P of [a, b] such that $0 \leq U(f,P) - L(f,P) < \epsilon$

(OR)

(b) State and prove fundamental theorem of calculus -I.

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