R-16

Code: 6308E2/19

Faculty of Science

B.Sc (Mathematics) III-Year, CBCS-VI Semester Examinations, May/June 2019

PAPER: VECTOR CALCULUS

Time: 3 Hours

Max Marks: 60

Section-A

I. Answer any Three of the following questions. (3x5=15 Marks)

1. Evaluate the line integral of the vector field $u = (xy, z^2, x)$ along the curve given by $x = 1 + t, y = 0, z = t^2, 0 \leq t \leq 3$
2. Evaluate surface integral of $u = (xy, x, x + y)$ over the surface S defined by $z = 0$ with $0 \leq x \leq 1, 0 \leq y \leq 2$, ,with the normal n directed in the positive z direction .
3. A cube $0 \leq x, y, z \leq 1$ has a variable density given by $\rho = 1 + x + y + z$ .Then find the total mass of the cube.
4. If the scalar field $f = xyz$, then find $grad(f)$.
5. Find the divergence of a vector field $u = (y, z, x)$
6. If the vector field $u = (xy, z + x, y)$. Then calculate $\nabla \times u$

Section-B

II. Answer the following questions. (3x15=45 Marks)

7. (a) Find the line integral of the vector field $u = (y^2, x, z)$ along the curve given by $z = y = e^x$ from $x = 0$ to $x = 1$.

(OR)

(b) If S is the entire x, y plane, evaluate the integral $I = \iint_S e^{-x^2-y^2} dS$, by transforming the integral into polar coordinates.

8. (a) Find the volume integral of the scalar field $\emptyset = x^2 + y^2 + z^2$ over the region V specified by $0 \leq x \leq 1, 1 \leq y \leq 2, 0 \leq z \leq 3$.

(OR)

(b) Find the directional derivative of the scalar field $f = 2x + y + z^2$ in the direction of the vector $(1,1,1)$, and evaluate at the origin .

9. (a) Find the Laplacian $\nabla^2\emptyset$ for the scalar field $\emptyset = x^2 + xy + y^2$

(OR)

(b) If c, d are scalars and u, v are vectors, then show that $\nabla \times (cu + dv) = c\,\nabla \times u + d\,\nabla \times v$

*****

R-16

Code: 6308E1/19

Faculty of Science

B.Sc (Mathematics) III-Year, CBCS-VI Semester Examinations, May/June 2019

PAPER: COMPLEX ANALYSIS

Time: 3 Hours   Max Marks: 60

Section-A

I. Answer any Three of the following questions. (3x5=15 Marks)

1. Show that the transformation $w = e^z$ maps the rectangular region $a \le x \le b$; $c \le y \le d$; on to the region $e^a \le \rho \le e^b; c \le \emptyset \le d$.
2. Use Cauchy-Riemann equations to the function $f(z) = z^2 = x^2 - y^2 + i2xy$, show that it is differentiable everywhere and that, $f'(z) = 2z$.
3. Evaluate $\int_c f(z)dz$, where $f(z) = \frac{z+2}{z}$ and c is the semi circle $z = 2e^{i\emptyset}(o \le \emptyset \le \pi)$.
4. Prove that $\left|\int_C \frac{Z+4}{Z^3-1} dz\right| \le \frac{6\pi}{7}$. Where C is the arc of the circle |z|=2 from z=2 to z=2i.
5. By using Cauchy's Integral formula, evaluate $\int_c \frac{e^{-z}dz}{z-\frac{\pi i}{2}}$ where C is the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2, y = \pm 2$.
6. State and prove Liouville's theorem.

Section-B

II. Answer the following questions. (3x15=45 Marks)

7. (a) Suppose that $f(z) = u(x,y) + iv(x,y); (z = x + iy)$ and $z_0 = x_0 + iy_0$; $w_0 = u_0 + iv_0$, then prove that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \to (x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y) \to (x_0,y_0)} v(x,y) = v_0$.

   (OR)

   (b) Derive Cauchy-Riemann Equations.

8. (a) Evaluate $\int_c f(z)dz$ where $f(z)$ is the branch $z^{-1+i} = \exp[(-1+i)\log z]$; $(|z| > 0, 0 < arg z < 2\pi)$ of the indicated power function C is the circle, $z = e^{i\theta}; (0 \le \theta \le 2\pi)$

   (OR)

   (b) Let C_R denote the upper half of the circle $|z| = R, (R > 2)$ taken in the counter clockwise direction, show that $\left|\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4} dz\right| \le \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}$ and hence show that the integral value is zero as $R \to \infty$

9. (a) If a function f is analytic throughout a simply connected domain D, then prove that $\int_c f(z)dz = 0$ for every closed contour C lying in D.

   (OR)

   (b) State and prove the fundamental theorem of Algebra.

*****