Code:50522/B

Faculty of Sciences

M.Sc (Mathematics) II-Semester Backlog Examinations, Jan-2024

Paper- II: ADVANCE REAL ANALYSIS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. If E_1 and E_2 are two measurable sets, then prove that $E_1 \cup E_2$ is measubale.
2. Let $\emptyset$ *and* ψ be simple functions which vanish outside a set of finite measure, the prove that $\int(a\phi + b\psi) = a\int\phi + b\int\psi$.
3. If f is a function of bounded variation on [a,b], prove that $f^1(x)$ exists a.e on [a.b].
4. Suppose A is a linear operator on R^n. Prove that A is invertible if and only if $\det|A| \neq 0$.
5. If $\{A_n\}$ is a countable collection of sets of real numbers then prove that $m^*(UA_n) \leq \sum M^*(A_n)$

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Prove hat outer measure of an interval is its length.

(OR)

(b) Let $E_1, E_2, \dots\dots\dots E_n$ be any finite disjoint measurable sets then for any set A prove that $m^*(A_n \cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} m^*(A_n E_i)$

7. (a) State and prove Fatous Lemma.

(OR)

(b) If f and g are bounded measurable functions defined on a set E of finite measure, then prove that $\int_E (af + bg) = a\int_E f + b\int_E g$.

8. (a) Prove that a function f is of bounded variation on [a,b] if and only if f is the difference of two monotone real valued function on[a,b].

(OR)

(b) If f is bounded and measurable on [a,b] and $F(x) = \int_a^x f(t)dt + F(a)$ then prove that $F^{\mathrm{l}}(x) = f(x)$ for almost all x in [a,b].

9. (a) Suppose f is defined in an open set $E \subseteq | R^2$. Suppose that $D_1 f, D_{21} f$ *and* $D_2 f$ exist at every point of E and $D_{21}, (f)$ is continuous at same point $(a,b) \in E$, then prove that $D_{12} f$ exists at (a,b) and $D_{12} f(a,b) = D_{21} f(a,b)$.

(OR)

(b) If [A] and [B] are n by n matrices then prove that det ([B][A]) = det[B] det[A].

10. (a) Prove that every Borel set is measurable.

(OR)

(b) State and prove Lebsgue dominated convergence theorem.

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Code:50523/BL

Faculty of Sciences

M.Sc (Mathematics) II-Semester Backlog Examinations, Jan-2024

Paper- III: FUNCTIONAL ANALYSIS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. If a normed space X has the property that the closed unit ball $M = \{x: \|x\| \leq 1\}$ is compact, then prove that X is finite dimensional.
2. Define orthonormal set in an inner product space. Prove that an orthonormal set is linearly independent.
3. Show that a norm on an inner product space satisfies the parallelogram equality $\|x+y\|^2 + \|x-y\|^2 = 2\,(\|x\|^2 + \|y\|^2)$.
4. Let X, Y be normed spaces and $S, T \in B(X,Y)$ Then prove that
   (i) $(S+T)^X = S^X + T^X$. (ii) $(\propto T)^x = \propto T^X \; \forall \, scalars \propto$.
5. IF H is a separable Hilbert space. Then prove that every orthonormal set in H is countable.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) (i) State and prove Riesz's lemma.

   (ii) Prove that on a finite dimensional vector space X, any norm $\|.\|$ is equivalent to any other norm $\|.\|_0$.

   (OR)

   (b) Prove that every finite dimensional subspace Y of a normed space X is complete and closed in X.

7. (a) Let X be an inner product space M a non-empty convex subset which is complete in the metric induced by the inner product. Then prove that for every given $x \in X$ there exists a unique $y \in M$ such that $\eth = Inf_{\tilde{y} \in M}\|x - \tilde{y}\| = \|x - y\|$.

   (OR)

   (b) State and prove Bessel's inequality.

8. (a) Let H_1 and H_2 be Hillbert spaces and T: $H_1 \rightarrow H_2$ be a bounded linear operator. Then prove that the Hillbert adjoint operator T^* of T exists, is unique and is bounded linear operator with norm$\|T^*\| = \|T\|$.

   (OR)

   (b) State and prove Riesz representation theorem for linear form.

9. (a) State and prove open mapping theorem.

   (OR)

   (b) State and prove Uniform boundedness theorem.

10.(a) Let $T: D(T) \rightarrow Y$ be a bounded linear operator with $P(T) \subset X$ where X,Y are normed spaces then

(i) If D(T) is closed subset of X then T is closed.

(ii) If T is closed and X is complete then D(T) is closed subset of X.

(OR)

(b) Prove that the vector space B(X, Y) of all bounded linear operators from a normed space X into a Banach space Y is itself a normed space with norm defined by

$$\|T\| = Sup_{x \in X_{x \neq 0}} \frac{\|Tx\|}{\|T\|} = Sup_{x \in X_{x=1}} \|T\|$$

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Code:50204/B

Faculty of Sciences

M.Sc (Mathematics) II-Semester Backlog Examinations, Jan-2024

Paper- IV: THEORY OF ORDINARY DIFFERENTIAL EQUATION

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. let f_1 and f_2 be linearly independent function on an interval I then prove that the functions $f_1+ f_2$ and $f_1- f_2$ are also linearly independent on I.
2. Find the particular solution by using the method of undetermined of coefficients $x^2 + 4x = 1 + 3t^2$
3. State and prove Grownwall Inequality
4. Find the Lipschitz constant and bound for $f(t,x) = e^t sinx, |x| \leq 2x \quad |t| \leq 1$ and also show that f(t,x) satisfies the lipscgitz condition in the rectangle indicated.
5. Solve IVP $x''' + x'' = 0,\ x(0) = 1,\ x'(0) = 0,\ x''(0) = 1$

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Solve $x''' + 7x' = (3 - 36t)e^{4t}$ using the mentod of undetermined coefficients.

(OR)

(b) State and prove the Abel's formula.

7. (a) If A(t) is an nxn matrix continuous on I and if a matrix ∅ satisfies $x^1 = A(t)x,\ t \in I$ then prove that det ∅ satisfies the equation $(det\emptyset)' = (trA)(det\emptyset)$.

(OR)

(b) Determine e^{tA} and a fundamental matrix for the system $X^1 = AX$ where
$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}.$$

8. (a) State and prove Picard's theorem.

(OR)

(b) Prove that IVP $x^1 = f(t,x), x(t_0) = x_0$ has a unique solution defined on $t_0 \leq t \leq t_0 + h$ if f is continuous in the strip $t_0 \leq t \leq t_0 + h,\ |x| < \infty$ and f satisfies Lipschitz condition $|f(t_1\, x_1) - f(t_1\, x_2)| \leq K|\, x_1 - x_2)|$, $k > 0$ by using contraction principle.

9. (a) Define upper and lower solution let $v, w \in c^1[t_0,\ t_0 + h]$ R be a lower and Upper solutions of $x^1 = f(t,x)\, x(t_0) = x_0$. Respectively suppose that for $x \geq y$ 'f' satisfies the inequality $f(t.x) - f(t,y) \leq L(x - y)$ where L is positive constant then $V(t_0) \leq w(t_0)$ implies that $v(t) \leq w(t),\ t \in [t_0,\ t_0 + h]$.

(OR)

(b) State and prove comparison theorem for IVP $x' = f(t,x),\ x(f_0) = x_0$.

10. (a) State Ascoli's lemma and also prove that existence of the unique solution of IVP$x = f(t,x),\ x(f_0) = x_0$ using Ascoli's lemma where 'f' is continuous and bounded on the strip $s: t_0 \leq t < t_0 + h,\ |x| < \infty$.

(OR)

(b)Show that the wronskians of the function $x_1(t),\ x_2(t), \dots\ x_n(t)$ defined on I is a non – zero if and only if the function $x_1(t),\ x_2(t), \dots\ x_n(t)$ are linearly independent.

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Code: 50531/R

Faculty of Sciences

M.Sc (Mathematics) III-Semester Regular Examinations, Dec-2023

Paper- I: Complex Analysis

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Show that Z and Z^1 Corresponds to diametrically Opposite points on the Riemann Sphere if $ZZ^{-1}=-1$.
2. State and Prove Luca's theorem.
3. Compute $\int xdz$ where ϑ is the directed line Segment from 0 to 1+i.
4. State and Prove the weistrass theorem on essential isolated Singularity.
5. Evaluate $\int_{|z|} z^n(1-z)^m \, dz$.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Show that the limit function of a uniformly Convergent Sequence of Continuous function is itself Continuous.

(OR)

(b) State and prove Sufficient Condition for analytic function.

7. (a) Define Cross ratio. Prove that Cross ratio is invariant under a linear trans formation.

(OR)

(b) Find the fixed points of linear transformation

W= $\frac{Z}{2z-1}$ and W= $\frac{2z}{3z-1}$

8. (a) State and Prove Cauchy's theorem from a rectangle.

(OR)

(b) Compute $\int_{|z|=1} \frac{e^2}{z} dz$.

9. (a) State and Prove Taylor's theorem.

(OR)

(b) Define Simply Connected region. Prove that a region Ω is simply Connected if and if $n(\vartheta, a) = 0$. Find all cycles $\vartheta \; in \; \Omega$ and all point 'a' which do not belong to Ω.

10. (a) Sate and Prove Scwartz lemma.

(OR)

(b) Compute $\int_{|z|=2} z^n(1-z)^m \, dz$ and $\int_{|z|=1} e^z z^{-n} \, dz$.

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Code: 50532/R

Faculty of Sciences

M.Sc (Mathematics) III-Semester Regular Examinations, Dec-2023

Paper- II: Elementary Operator Theory

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Let 'X' be a finite dimensional Inner Product Space and T:$X \to X$ is a linear operator. If 'T' is self adjoint then show that it's Spectrum is real.
2. Prove that every linear operator on a finite dimensional normed space is compact.
3. If T:$H \to H$ is a bounded self adjoint linear operator on a complex Hilbert space H, then prove that all the eigen values of T are real.
4. For any projection 'P' on a Hilbert space H, prave that $< px, x >= \|PX\|^2$, P≥0, $\|PX\| \leq 1$
5. Let X be complex Banach space, $T \epsilon B\ (x, x)$ and $\lambda, \mu \epsilon R(T)$ then prove that $R_\mu - R_\lambda = (\mu - \lambda) R_\mu\, R_\lambda$

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Sate and Prove the Spectral mapping theorem for polynomials.

(OR)

(b) Prove that the resolvent set ρ (T) of a bounded linear operator T on a complex Banach space X is open.

7. (a) Let B be a subset of a metric space X. then prove that
   (i) If B is relatively compact, B is totally bounded.
   (ii) If B is totally bounded and X is compact, B is relatively compact.

(OR)

(b) Let T:$X \to X$ be a compact linear operator on a normed space X then prove that for every $\lambda \neq 0$ the range of $T_\lambda = T - \lambda T$ is closed.

8. (a) λ Prove that the residual spectrum $\sigma_{r\,(T)}$ of a bounded self adjoint linear operator T:$H \to H$ on a complex Hilbert space H is empty.

(OR)

(b)If two bounded self adjoint linear operators S and T on a Hilbert space H are positive and commute then prove that their product ST is positive.

9. (a) A bounded linear operator P: $H \to H$ on a Hilbert space H is a projection if and only if P is self adjoint and Idempotent.

(OR)

(b) State and Prove Positive square root theorem.

10. (a) State and Prove Inverse theorem.

(OR)

(b) Let T be a bounded self adjoint linear operator on a complex Hilbert space H and $m =_{llxll=1}^{inf<Tx,x>}$, $M =_{llxll=1}^{Sup<Tx,x>}$ then prove that m and M are . spectral values of T.

Code: 50533/R

Faculty of Sciences

M.Sc (Mathematics) III-Semester Regular Examinations, Jan-2024

Paper- III: Operations Research

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Explain the problem of LPP in general. Define feasible solution optimum solution of LPP.
2. Find the Inverse of a matrix $\begin{bmatrix} 4 & 1 \\ 2 & 9 \end{bmatrix}$ by simplex method.
3. Explain briefly about the formulation of assignment problem.
4. Explain about the characteristics of dynamic programming problem.
5. Explain Matrix Minima method of solving a transportation problem.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Solve by Simplex method $Max\ Z = x_1 + x_2 + x_3$, subject to $4x_1 + 5x_2 + 3x_3 \leq 15$, $10x_1 + 7x_2 + x_3 \leq 12$ and $x_1, x_2, x_3 \geq 0$

(OR)

(b) Solve the following LPP; MinZ= $600x_1 + 500x_2$, subject to $2x_1 + x_2 \geq 80$, $x_1 + 2x_2 \geq 60$; $x_1, x_2 \geq 0$. Using Big-M method.

7. (a) Write the steps in the dual Simplex algorithm for solving LPP.

(OR)

(b) Solve the following LPP by dual simplex method, $minZ = x_1 + x_2$ subject to $2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$ and $x_1, x_2 \geq 0$.

8. (a) Solve the Transportation problem.

	D_1	D_2	D_3	
O_1	3	10	1	50
O_2	8	12	9	45
O_3	7	7	3	85
	50	60	70	180

(OR)

(b) Solve the Assignment problem.

	M_1	M_2	M_3	M_4
J_1	5	8	3	2
J_2	4	10	12	4
J_3	8	6	9	4
J_4	3	5	9	8

9. (a) Use dynamic Programming to show that $\sum_{i=0}^{n} P_i log P_i$ subject to $\sum_{i=1}^{n} P_i = 1$ is minimum when $P_1 = P_2 = - - - -= P_n = 1/n$

(OR)

(b) Use the principle of optimality and find $Max\ Z = b_1x_1 + b_2x_2 + \cdots + b_nx_n$ when $x_1 + x_2 + \cdots + x_n = c$ and $x_1, x_2, \ldots . . x_n \geq 0$ $b_1 > 0, b_2 > 0 \ldots . b_n > 0$

10.(a) Solve the following assignment problem and obtain optimum solution.

Employes

Jobs	A	B	C
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

(OR)

(OR)

(b) Using dynamic programming find $Max\ Z = y^2{}_1 + y^2{}_2 + y^2{}_3$ subject to $y_1, y_2, \ldots . y_n = c, c \neq 0$ and $y_i \leq 0; i = 1,2, \ldots n$

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Code:50534/R

Faculty of Sciences

M.Sc (Mathematics) III-Semester Regular Examinations, Jan-2024

Paper- IV: INTEGRAL EQUATIONS

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Verify $\emptyset(x) = 3$ is a solution of $x^3 = \int_a^x (x-t)^2 \emptyset(t)dt.$
2. Solve the integral equation by using Laplace transformation $\int_a^x e^{x-t}\emptyset(t)dt = x^2.$
3. Construct resolvent kernel for the kernal $k(x,t) = e^{x+t}$
4. Define Green's function with four property.
5. Find the characteristic number for the following integral equation $\emptyset(x) = \lambda \int_0^\pi Cos(x+t)\phi(t)dt = 0$.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Using the method of successive approximation, solve the integral equation $\emptyset(x) = 1 + \int_0^\pi Cos(x-t)\phi(t)dt, \phi_0(x) = 1.$

(OR)

(b) Solve the integral equation $\emptyset(x) = \frac{1}{1+x^2}\int_0^x Sin(x-t)\phi(t)dt.$

7. (a) Solve the integral equation by using Laplace transformation $\phi(x) = Cosx + \int_0^\infty e^{x-t}\emptyset(t)dt$

(OR)

(b) Solve the integral equation $\int_0^x (x-t)^2 \emptyset(t)dt = x^2 + x^3.$

8. (a) Solve the integral equation $\emptyset(x) - 4\int_0^{\pi/2} Sin^2(x)\,\emptyset(t)dt = 2x - \pi$ with degeneration kernel

(OR)

(b) Using the resolvent kernel, Solve the integral equation $\emptyset(x) - \int_0^{2\pi} Sinx\, Cost\, \emptyset(t)dt = Cos2x$

9. (a) Construct the Green's function for the $B \curlyvee P$ $Y^{Iv}(x) = 0,\ y(0) = y^1(0)\ and\ y(1) = y^1(1) = 0$.

(OR)

(b) Solve the homogeneous integral equation $\emptyset(x) - \lambda\int_0^{2\pi} Sin(x+t)\,\emptyset(t)dt = 0$

10.(a) Convert the boundary value problem to integral equation $Y'' - \lambda y = e^x,\ y(0) = y^1(0),\ y(1) = y^1(1)$

(OR)

(b)Solve the integral equation $\emptyset(x) = \lambda\int_0^1 xt\,\emptyset^2(t)dt$ where λ is a parameter.

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Code:50535/R

Faculty of Sciences

M.Sc (Mathematics) III-Semester Regular Examinations, Jan-2024

Paper- V: NUMERICAL TECHNIQUES

Time: 3 Hours **Max Marks: 70**

Section-A

I. Answer the following questions in not more than ONE page each (5x4=20 Marks)

1. Find real root of the equation $x^3+x-1=0$ by using Bi section method.
2. Solve the equations $10x_1-x_2+2x_3=4$
   $X_1+10x_2-x_3+3$
   $2x_1+3x_2+20x_3=7$ Using the Gauss elimination method
3. Obtain the piece wise(lagranges) linear interpolating polynomials for the function f(x) defined by the data

x	1	2	4	8
f(x)	3	7	21	73

   Estimate the value of f(3).
4. Evaluate $\int_0^1 \frac{1}{1+x}dx$ by using Simpson's 1/3 rule with h=0.5.
5. Find the real root of the equation $x^3-2x-5=0$ by using Newton-Raphson method.

Section-B

II. Answer the following questions in not more than FOUR pages each (5x10=50 Marks)

6. (a) Explain the Newton-Raphson method to find the real root of the equation$x=e^{-x}$.

   (OR)

   (b) Perform three iterations of the Muller method to find the smallest possible root of the equation $f(x)= x^3-5x+1=0$.

7. (a) Solve the equation 3x+y+2z=3
   2x-3y-z=-3
   X+2y+z=4 by matrix inversion method.

   (OR)

   (b) Solve the equation 2x+3y+z=9
   X++2y+3z=6
   3x+y+2z=8 by the faxtorization method.

8. (a) Construct the divided difference table for the dat a

x	0.5	1.5	3.0	5.0	6.5	8.0
f(x)	1.625	5.875	31.0	131.0	282.125	521.0

   Find the interpolating polynomial and an approximate to the value of f(7).

   (OR)

   (b) Find the least squares approximation of second degree for the following data

x	-2	-1	0	1	2
f(x)	15	1	1	3	19

9. (a) From the Taylor series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y' = x - y^2$ and y(0)=1.

(OR)

(b) Using Runge-Kutta fourth order method find y(0.1) and y(0.2) given that $\frac{dy}{dx} = y - x$ and y(0)=2.

10. (a) By using Simpsons 3/8 rule to obtain the value of $\int_0^{0.3}(1 - 8x^3)dx$ with h=0.1.

(OR)

(b) Solve the system of equation $4x_1 + x_2 + x_3 = 2,\ x_1 + 5x_2 + 2x_3 = -6,$ $x_1 + 2x_2 + 3x_3 = -4$ using Gauss-Seidel method..

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