Code No. 1811

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2016

ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Suppose X is a G-set, where G is a group. Show that the power set p(X) of X is also a G-set.
2. Let G be a group of order pq, where p and q are primes such that p>q and $q \nmid (p-1$. Then Prove that G is cyclic.
3. Prove that the ring $\mathbb{Z}[i]$ of Gaussian integers is a Euclidean domain.
4. Suppose R is a commutative integral domain with unity. Prove that every prime in R is irreducible.
5. Suppose R is a Euclidean domain. Show that $a \in R$ is a unit $\Leftrightarrow \varphi(a) = \varphi(1)$, where φ is the function mentioned in the definition of Euclidean domain.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Suppose G is a solvable group. Then prove that every subgroup of G and every homomorphic image of G are solvable.

(OR)

b) Suppose G is a group. X is a G-set. Then prove that
i) the set of all orbits in X is a partition of X.
ii) $|Gx| = [G: G_x]$ for each $x \in X$
iii) If $|x| < \infty$, then $|x| = \sum_{x \in C}(G: G_x)$, where C is a subset of X containing exactly one element from each orbit.

7. a) State and prove First Sylow theorem.

(OR)

b) Let A be a finite abelian group. Then show that there exists a unique list of integers $m_1, m_2, \ldots\ldots\ldots m_k$ (all > 1) such that $|A| = m_1.m_2 \ldots\ldots m_k$, $m_1/ m_2/ m_3/\ldots\ldots/m_k$ and $A = C_1 \oplus C_2 \oplus \ldots\ldots\ldots\ldots \oplus C_k$ where C_i are cyclic sub groups of A of order m_i.

8. a) Let R be a non zero ring with unity and I is an ideal in R such that $I \neq R$. Then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

(OR)

b) Suppose $A_1, A_2, \ldots\ldots\ldots, A_n$ are right ideals in a ring R. Then prove that the following are equivalent.
i) $A = \sum_{i=1}^{n} A_i$ is a direct sum.
ii) $o = \sum_{i=1}^{n} a_i, a_i \in A_i \Rightarrow a_i = 0, i = 1,2, \ldots\ldots, n$
iii) $A_i \cap \sum_{\substack{j=1 \\ j \neq 1}}^{n} A_j, = (0), j = 1,2, \ldots\ldots, n$

P.T.O.

9. a) Let $\{N_i\}_{i\in\Delta}$ be a family of R-sub modules of an R-module M. Then prove that the following are equivalent.
i) $\sum_{i\in\Delta} N_i$ is a direct sum.
ii) $o = \sum_{i\in\Delta}^{n} x_i, x_i \in N_i \Rightarrow x_i = 0, \forall_i$
iii) $N_i \cap \sum^{n}_{\substack{j\in\Delta \\ j\neq i}} N_j, = (0), \forall_i \in \Delta$

(OR)

b) State and prove Schur's lemma.

10. a) Suppose R is a commutative ring, P an ideal in R. Then prove that P is a prime ideal $\Leftrightarrow$ $ab\in P, a\in R, b\in R \Longrightarrow a\in P$ or $b\in P$.

(OR)

b) Suppose R is a nonzero commutative ring with unity. Let M be an ideal in R, $M \neq R$. Then prove that M is a maximal ideal in R $\Leftrightarrow \frac{R}{M}$ is a field.

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Code No. 1811

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2017

ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Define G-set and give an example.
2. Prove that there are no simple groups of order 63.
3. For any two ideals A and B in a ring R prove that: $\frac{A+B}{B} \cong \frac{A}{A\cap B}$.
4. Show that the group $\frac{z}{(10)}$ is a direct sum of $H = \{\bar{0}, \bar{5}\}$ and $k = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$.
5. Prove that every Euclidean domain is PID.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) If G is a nilpotent group then prove that every subgroup of G and every homomorphic image of G are nilpotent.

(OR)

b) State and prove first Isomorphism theorem.

7. a) State and prove second and third Sylow theorem.

(OR)

b) If A and B be R-sub modules of R-modules M and N then show that: $\frac{M\times N}{A\times B} \cong \frac{M}{A} \times \frac{N}{B}$.

8. a) State and prove Jordan-Holder theorem.

(OR)

b) Show that every PID is a UFD, but a UFD is not necessarily a PID.

9. a) Find the rank of the linear mapping:
$\emptyset: \mathbb{R}^4 \to \mathbb{R}^3$ Where
$\emptyset(a, b, c, d) = (a + 2b - c + d, -3a + b + 2c - d, -3a + 8b + c + d)$.

(OR)

b) For any two ideals A and B in a ring R, prove that:

$$\frac{A+B}{A\cap B} \cong \frac{A\times B}{A} \times \frac{A+B}{B} \cong \frac{B}{A\cap B} \times \frac{A}{A\cap B}$$

10. a) State and prove fundamental theorem of R-Homomorphism.

(OR)

b) Show that the commutative integral domain $R = \{a + b\sqrt{-5} \,/ a, b \in z\}$ is not a unique factorization domain.

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Code No. 1811

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, JAN-2019

ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Prove that the mapping $\sigma(x) = x^m$ is an automarphism of a finite abelian group G of order n and (m, n) =1.
2. Prove that a group of order 42 is not simple.
3. Show that the Kernel of a homomorphism of a ring R into another ring R is an ideal of R.
4. Let V be a vector space of differentiable functions from R to R. Let D be the differential operator on R. Find the matrix corresponding to the matrix $B = \{1, x, x^2\}$.
5. Applying sylow theorem show that if P divides |G| then G has an element of order P.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Let G be a group active on a set X then for $x \in X$. Show that $|Gx| = [G: G_x]$where G_x is the orbit of x in G and G_x is the isotropy subgroup of x in G.

(OR)

b) Let G be a nilpotent group. Then show that every subgroup of G and every homomorphic image of G are nilpotent.

7. a) Show that any two sylow P subgroups of a finite group G are conjugate.

(OR)

b) Let G be a group of order pq, where p and q are primes such that p>q and q ∤ (p-1), then show that G is cyclic.

8. a) State and Prove Chinese Remainder theorem of Rings.

(OR)

b) Show that every Euclidean domain is a PID.

9. a) Let R be an integral domain. Show that R is a right ore domain if and only if there exist a division ring Q such that.

i) R is a sub ring of Q

ii) Every element of Q is of the form ab^{-1} for some a, b ∈ R.

(OR)

b) Let R be a ring with unity. Show that R is isomorphic to Hom (R,R) where Hom(R,R) is the ring of endomorphism of R.

10. a) State and Prove Caley's theorem.

(OR)

b) If R is a ring with 1 and I is an ideal of R such that I ≠ R then show that there exists a maximal ideal M in R Containing I.

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Code No. 1811

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, FEB-2020

ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Define a normal subgroup. Give one example.
2. Prove that there is no simple group of order 63.
3. Define a left ideal, right ideal and give an example for each.
4. Give two examples of sub modules with brief explanations.
5. Prove that every group order p^2 (p is prime) is abelian.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) State and prove cayley's theorem

(OR)

b) State and prove Burnside theorem.

7. a) State and prove sylows first theorem

(OR)

b) If G is a group of order pq, where p and q are prime numbers such that $p > q$ and q does not divide $p - 1$ then prove that G is cyclic.

8. a) Define a maximal ideal. Prove that an ideal M of a commutative ring R with unity is maximal $\Leftrightarrow R/M$ is a field.

(OR)

b) Define PID and UFD. Prove that every PID is a UFD. Is converse true? Justify.

9. a) If M is a finitely generated free module over a commutative ring R. Then prove that all bases of M have the same number of elements.

(OR)

b) Let R be a UFD and let S be a multiplicative subset of R containing the unity of R. Then prove that RS is also a UFD.

10. a) Define a solvable group. Prove that a group G is solvable $\Leftrightarrow$ G has a normal series with abelian factors. Further a finite group is solvable $\Leftrightarrow$ its composition factors are cyclic groups of prime orders

(OR)

b) Define a prime idal. Give an example. Prove that an ideal P of a commutative ring R with unity is prime $\Leftrightarrow$ R/P is an integral domain.

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Code No. 1811

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY-2022

ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. State and prove Burnside theorem.
2. Show that a group of order 36 is not simple.
3. If R is a ring with unity then show that each maximal ideal is prime ideal.
4. Show that any commutative integral domain R can be embedded in a field R_S.
5. Show that an irreducible element in a commutative principal ideal domain is always prime.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Let G be group acting on a set X. Then show that the set of all orbits in X under G is a partition of X and for any $x \in X$ there is a bijection $Gx \to \frac{G}{G_x}$ and hence $|Gx| = [G: G_x]$.therefore if X is finite then show that $|X| = \sum_{x \in C} [G: G_x]$ where C is any subset of X containing exactly one element from each orbit.

(OR)

b) State and prove Jordan-Holder theorem.

7. a) State and prove Fundamental theorem of finitely generated abelian groups.

(OR)

b) State and prove 2^{nd} and 3^{rd} Sylow theorem.

8. a) If a ring R has unity then show that every ideal I in the matrix ring R_n is of the form A_n where A is an ideal of R.

(OR)

b) Show that the product of two primitive polynomials is primitive.

9. a) Let R be a UFD and S be a multiplicative subset of R containing the unity of R. Then show that R_S is also UFD.

(OR)

b) State and prove the Fundamental theorem of R-homomorphism.

10. a) Let G be nilpotent group. Then show that every subgroup of G and every homomorphic image of G is nilpotent.

(OR)

b) State and prove Schur's lemma.

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Code No. 1812

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2016

REAL ANALYSIS

PAPER – II

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Define rearrangement of a series of numbers. Give an example to show that rearrangement of a convergent series need not be a convergent series.
2. With usual notation prove that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$.
3. Prove that a sequence $\{f_n\}$ convergence to f with respect to the metric of C(X) if and only If $f_n \to f$ uniformly on X.
4. Suppose f maps a convex open set $E \subset R^n$ into R^m, f is differentiable on E and there exists A real number M such that $||f^1(x)|| \leq M$ for every $x \in E$. Prove that $|f(b) - f(0)| \leq M|b - a|$ for all $a \in E, b \in E$.
5. Define contraction mapping and give an example. Prove that every contraction mapping is continuous.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Define upper and lower limits of a sequence $\{s_n\}$. Prove that
   i) $s^* \in E$
   ii) If $x > s^*$ there exists an integral such that $s_n < x$ whenever $n \geq$ N. Moreover prove that s^* is the only number with the properties (i) and (ii).

   (OR)

   b) i) Prove that monotonic functions can not move discontinuities of second kind.
   ii) Prove that the set of discontinuities of a monotonic function f on (a, b) is at most countable.

7. a) Prove that $f \in R(\alpha)$ on [a, b] if and only if for every $\epsilon > 0$ there exists a partition P of [a, b] such that $U(p, f, \alpha) - L(p, f, \alpha) < \epsilon$.

   (OR)

   b) i) If $f \in R(\alpha)$ on [a, b] if $|f(x)| \leq M$ on [a, b] prove that $|\int_a^b f d\alpha| \leq M(\alpha(b) - \alpha(a))$.
   ii) State and prove the fundamental theorem of Calculus.

8. a) Suppose $\{f_n\}$ is a decreasing sequence of continuous functions defined on a compact space K, which conversion point wise to a limit function f which is continuous on K prove that $f_n \to f$ uniformly on K.

   (OR)

   b) Suppose f is a continuous complex value function defined on [a, b]. Prove that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on [a, b].

P.T.O.

9. a) Suppose X is a vector space of dimension on prove that following
   i) A set E of n vectors in X spans X if and only if E is independent.
   ii) X has a basis and every basis consists of n vectors.
   iii) if $1 \leq r \leq n$ and $\{y_{1,} y_{2,} \dots \dots y_r\}$ is an independent set in X then X has a basis continuing $\{y_{1,} y_{2,} \dots \dots y_r\}$

(OR)

   b) Define a fixed point. Prove that every contraction mapping defined on a complete metric space has a unique fixed point.

10. a) Suppose $f_n \to f$ uniformly on a set E in a metric space x and x is a limit point of E. Prove that $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{n\to\infty} \lim_{t\to x} f_n(t)$.

(OR)

   b) Define $C(x)$ and supremum norm on it. Prove that $C(x)$ is a complete metric space with respect to the metric induced by the supremum norm.

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Code No. 1812

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2017

REAL ANALYSIS

PAPER – II

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Show that Cauchy Product of two Convergent Series need not be Convergent.
2. *Let* $\{S_n\}$ be a sequence of real numbers and E be the set of all sub sequential limits in the extended real number system of $\{S_n\}$ and S^*sup E *Prove that* $S^* \in E$.
3. Suppose $f \in R(\alpha)$ and $g \in R(\alpha)$ an [a, b]. Prove that fg and |f| belongs to $R(\alpha)$ on [a, b] also prove that $|\int_a^b f dx| \leq \int_a^b |f| dx$.
4. Define uniform Convergence of a series of functions. Give an example of a series of continuous function where sum function is not continuous.
5. Suppose X is a vector space of dimension n. Prove that a set E of n vectors in X Spans X if and only if E is independent.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) State and Prove Riemann's theorem on rearrangement of series.

(OR)

b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X prove that f(E) is a connected subset of Y.

7. a) i) Suppose f is continuous on [a, b] Prove that $f \in R(\alpha)$ on [a, b].
ii) If f is monotonic on [a, b] and α is continuous on [a, b] prove that $f \in R(\alpha)$ *on* $[a, b]$.

(OR)

b) If f is a bounded function defined on [a, b] and if α is monotonically increasing and differentiable an [a, b] such that α^1 is Riemann integrable on [a, b] then prove that $f \in R(\alpha)$ an [a, b] if and only if $f\alpha^1$ is Riemann integrable an [a, b] and also prove that $\int_a^b f dx = \int_a^b f\alpha^1 \, dx$.

8. a) Suppose $\{f_n\}$ is a sequence of continuous functions defined on a compact set K. such that $\{f_n\}$ Converges point wise to a continuous function f on K and if $f_n(x) \geq f_{n+1}(x)$ $\forall x \in K,\ n = 1, 2, 3, \ldots..$ Prove that $f_n \to f$ uniformly on K.

(OR)

b) State and Prove Cauchy's Criterion for uniform convergence of a sequence of functions.

9. a) i) *if* $A \in \Omega$ and $B \in L(R^n)$ and $||B - A||\ ||A^{-1}|| < 1$ then $B \in \Omega$.
ii) Ω is an open subset of $L(R^n)$ and the mapping $A \to A^{-1}$is continuous on Ω.

(OR)

b) Let f maps an open set $E \leq R^n$ into R^m. Then prove that f is continuously differentiable on E if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m$ *and* $1 \leq j \leq n$.

P.T.O

10. a) If f is a bounded function defined on [a, b] and f has finite number of points of discontinuities and if α a increasing function which is continuous at the points where f is discontinuous then prove that $f \in R(\alpha)\ on\ [a, b]$.

(OR)

b) Suppose E is an open set in R^n. f maps E into R^m f is differentiable at $x_0 \in E$. g maps an open set containing $f(E)$ into R^K and g is differentiable at $f(x_0)$ then prove that the mapping F if E into R^K defined by $F(x) = g(f(x))$ is differentiable at x_0 and $F^1(x_0) = g^1(f(x_0))\, f^1(x_0)$

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Code No. 1812

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, JAN-2019

REAL ANALYSIS

PAPER – II

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. If $|a_n| \leq c_n$ for all $n \geq N_0$ where N_0 is some fixed integer and if $\sum c_n$ Converges then show that $\sum a_n$ Converges.
2. If P^* is any refinement of P then show that $U(P^*.f.\alpha) \leq U(P.f.\alpha)$
3. When do you say that $< f_n >$ is convergent on E? When do you say that $< f_n >$ is uniformly convergent on E? What is the relation between convergence and uniform convergence?
4. When do you say that a linear operator is invertible? If A is a linear operator on X then show that A^{-1} is linear.
5. Define Riemann-Stieltjes integral as limit of sum.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Let f be a continuous mapping of a compact metric space X into a compact metric space Y. Then show that f is uniformly continuous on X.

(OR)

b) Suppose that f is a continuous 1-1 mapping of a compact metric space X onto a metric Space Y. then show that the inverse mapping $f^{-1}: Y \rightarrow X$ is a continuous mapping.

7. a) If f is monotonic on [a,b] and if α is continuous on [a,b] then show that $f \in R(\alpha)$.

(OR)

b) Suppose that $f \in R(\alpha)$ on [a, b], $m \leq f \leq M$, Φ is continuous on $[m, M]$ and $h(x) = \Phi(f(x))$ on [a,b]. Then show that $h \in R(\alpha)$ on [a,b].

8. a) Suppose that $f_n \rightarrow f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\underset{t \to x}{Lt} f_n(t) = A_n$ $(h = 1,2,3, \dots\dots)$ then $< A_n >$ converges and $\underset{t \to x}{lim} f(t) = \underset{n \to \infty}{lim} A_n$.

(OR)

b) Let α be monotonically increasing on [a,b]. Suppose that $f_n \in R(\alpha)$ on [a,b] for n=1,2,3…. and suppose that $f_n \rightarrow f$ uniformly on [a,b]. Then show that $f \in R(\alpha)$ on [a,b] and $\int_a^b f d\alpha = \underset{n \to \infty}{lim} \int_a^b f_n d\alpha$

9. a) Let r be a positive integer. If a vector space X is spanned by a set of r vectors then show that dim X$\leq$ r.

(OR)

b) Show that a linear operator A on a finite dimensional vector space Y is one-to-one if and only if the range of A is all of X.

10. a) If any metric space X show that every convergent sequence is a Cauchy sequence.

(OR)

b) If $A, B \epsilon L(R^n . R^m)$ and c as scalar then show that

$||A + B|| \leq ||A|| + ||B||$

$||cA|| \leq |c| ||A||$.

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Code No. 1812

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY-2022

REAL ANALYSIS

PAPER – II

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. If $\sum a_n = A$ and $\sum b_n = B$, then prove that $\sum (a_n + b_n) = A + B$ and $\sum ca_n = cA$ for any fixed c.
2. If f is continuous on [a,b], then prove that $f \in R(\alpha)$ on [a,b].
3. Prove that $f_n(x) = \frac{x}{1+nx^2}$ on [a,b] is uniformly convergent.
4. Let Ω be the set of all invertible operators on R^n, If $A \in \Omega$, $B \in L(R^n)$ and $\|B - A\| \|A^{-1}\| < 1$, then prove that $B \in \Omega$.
5. If $f, g \in R(\alpha)$ on [a,b],then prove that $fg \in R(\alpha)$ on $[a,b]$.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Prove that continuous image of a compact metric space is compact.

(OR)

b) If f is a continuous function of a compact metric space X into a metric space Y, then prove that f is uniformly continuous on X.

7. a) Suppose $f \in R(\alpha)$ on [a,b], $m \le f \le M$, ϕ is continuous on [m, M] and $h(x) = \phi(f(x))$ on [a,b]. Then prove that $f \in R(\alpha)$ on [a,b].

(OR)

b) If $f \in R(\alpha)$ on [a,b] and if $|f(x)| \le M$ on [a,b], then prove that $\left|\int_a^b f\, d\alpha\right| \le M[\alpha(b) - \alpha(a)]$

8. a) Suppose $\{f_n\}$ is sequence of functions and differentiable on [a,b] such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f_n^1\}$ converges uniformly on [a,b]. Then prove that $\{f_n\}$ converges uniformly on [a,b] to a function f and $f^1(x) = \underset{n\to\infty}{Lt} f_n^1(x)$.

(OR)

b) Suppose $\{f_n\}$ is sequence of continuous functions on E and if $f_n \to f$ uniformly on E, then prove that f is continuous on E. .

9. a) prove that a linear operator A on a finite-dimensional vector space X is one- to-one if and only if the range of A is all of X.

(OR)

b) State and prove Inverse function theorem.

10. a) If γ is continuous on [a,b] then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma^1(t)|\, dt$

(OR)

b) State and prove Stone weierstrass theorem

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Code No. 1813

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2016

TOPOLOGY

PAPER – III

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Suppose X is a topological space. Prove that any closed subset of X is the disjoint union of its set of isolated points and its set of limit points.
2. Suppose X is a topological space and $\{X_i\}$ is a non empty finite clan of compact subspaces of X show that $\cup_i X_i$ is also a compact sub space of X.
3. Deduce Urysohn's lemma from Tictze's extension theorem.
4. Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X on to the discrete two point space {0, 1}.
5. Suppose X is any arbitrary non-empty set and S is any arbitrary clan of subsets of X. Prove that S can serve as an open sub base for a topology on X.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) State and prove Kuratowski closure axioms.

(OR)

b) In a second countable space if a non-empty open set G can be written as a union of a class $\{G_i\}$ of open sets prove that G can be written as a countable union of G_i s.

7. a) State and prove Lebesgue covering lemma.

(OR)

b) Prove that every sequentially compact metric space is
i) Totally bounded ii) Compact

8. a) i) Prove that every compact Hansdorff space is normal.
ii) Show that a closed subspace of a normal space is normal.

(OR)

b) State and prove Urysohn's imbedding theorem.

9. a) i) Prove that every interval of R is a connected set.
ii) Suppose A is a connected sub space of a topological space X and B is a subspace of X such that $A \leq B \leq \overline{A}$. Prove that B is connected.

(OR)

b) Define a component of a topological space. State and prove any three main facts about components.

10. a) State and prove Urysohn's lemma.

(OR)

b) Prove that a subset of R is connected if and only if it is an interval.

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Code No. 1813

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2017

TOPOLOGY

PAPER – III

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Suppose A and B are arbitrary subsets of a topological space, prove the following:
   $i)\ \overline{\emptyset} = \emptyset \quad ii)\ \bar{A} \subseteq \bar{A} \quad iii)\ \bar{\bar{A}} = \bar{A} \quad iv)\ \overline{A \cup B} = \bar{A} \cup \bar{B}$
2. State and prove Lindelof's theorem.
3. Define T_1 and T_2 spaces. Give an example of a space which is T_1 but not T_2 with justification.
4. Show that any continuous image of a connected space is connected.
5. Show that every compact metric space has the Balzano-Weistrass property.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) State and prove Kuratowski closure axioms.

   (OR)

   b) i) Prove that every separable metric space is second countable.
   ii) Let X be a topological space and A an arbitrary sub-set of X. Then show that $\bar{A}$ ={X/each neighborhood of X intersects A}.

7. a) State and prove Ascalli's theorem.

   (OR)

   b) i) Prove that any continuous function from a compact metric X space into a metric space Y is uniformly continuous.
   ii) Prove that every sequentially compact metric space is totally bounded.

8. a) State and prove Lebesgue Covering Lemma.

   (OR)

   b) i) Prove that every sequentially copact metric space is compact.
   ii) Prove that if X is compact metric space then X is separable.

9. a) State and prove Urysohn's Lemma.

   (OR)

   b) i) Let X be a topological space and A be connected sub space of X. If B is a sub-space of X such that $A \subseteq B \subseteq \bar{A}$ then prove that B is connected.
   ii) Prove that a topological space is disconnected if and only if there exists a continuous mapping of X onto the discrete two-point space $\{0,1\}$.

10. a) Define a connected space. Prove that a sub-space of the real line R is connected if and only if it is an interval.

   (OR)

   b) State and prove Tietze Extension theorem.

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Code No. 1813

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, FEB-2019

TOPOLOGY

PAPER – III

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Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Let $\bar{A}$ be the closure of a subset A in a topological space X. Show that $\overline{A \cup B} = \bar{A} \cup \bar{B}$
2. Show that every compact metric space has the Balzano-Weoersirass property.
3. Show that a topological space is a T_1 space if and only if each point is a closed set.
4. Define a connected space. Give an example of a disconnected topological space.
5. Show that the real line R is separable.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Let X be a topological space and A is an arbitrary subset of X. show that $\bar{A} = \{x: \text{each neighborhood of x intersects A}\}$

(OR)

b) Let $f: X \rightarrow Y$ be a mapping between two topological spaces X and Y, show that
i) f is continuous $\Leftrightarrow$ inverse image of each basic open set is open.
ii) f is open $\Leftrightarrow$ the image of each basic open set is open.

7. a) Show that every closed subspace of a compact space is compact.

(OR)

b) Show that every sequentially compact metric space is totally bounded.

8. a) Show that a one to one continuous mapping of a compact space onto a Hausdorff space is a Homeomorphism.

(OR)

b) Prove that every compact Hausdorff space is normal.

9. a) Show that the open interval (a, b) of real line is connected.

(OR)

b) State and prove Tychonoff's theorem.

10. a) State and prove Heine Borel theorem of real line.

(OR)

b) Let X be a topological space and A is a connected sub space of X. If $A \subseteq B \subseteq \bar{A}$. Then prove that B is connected.

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Code No. 1813

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY-2022

TOPOLOGY

PAPER – III

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Suppose (X, τ) is a topological space and Y is a subset of X. Then $\tau_Y = \{G \cap Y: G\epsilon\tau\}$ is a topology giving the topological space (Y, τ_Y)
2. Prove that continuous image of compact toplogical space is compact.
3. Define completely regular toplogical space and normal topological space.
4. Define a Product topological space.
5. A topological space X is a T_1 – space if and only if each point of it is closed.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) suppose (X, τ) is a topological space and E is a subset of X then prove that $\bar{E} = \{x \in X: Every\ nbd\ of\ x\ intersects\ with\ E\}$

(OR)

b) Show that every separable metric space is second countable.

7. a) state and prove Lebesgue covering lemma.

(OR)

b) A metric space is sequentially compact if and only if it has the Bolzano Weirstrass property.

8. a) Suppose X is a T1- space then X is normal if and only if every nbd of a closed sets F has a nbd whose closure lies in the nbd.

(OR)

b) State and Prove Uryshon's lemma.

9. a) Suppose X is a topological space let $\{A_\alpha\}_{\alpha\epsilon\Delta}$ be a non empty class of non empty connected subsets of X such that $\bigcap A_\alpha$ where $\alpha\epsilon\Delta$ is non empty then $\bigcup A_\alpha$ is also connected in X.

(OR)

b) The product topological space of any non empty class of connected spaces is also connected.

10. a) Suppose X and Y are topological spaces then a mapping f: X→Y is continuous on X if and only if $f(\bar{A}) \subseteq \overline{f(A)}$ for every A⊆ X.

(OR)

b) Suppose A is connected subspace of a topological space and B is a subset of X such that $A \subseteq B \subseteq \bar{A}$ then B is also connected in particular $\bar{A}$ connected whenever A is.

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Code No. 1814

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2016

ELEMENTARY NUMBER THEORY

PAPER – IV

Time: 3 Hours] [Max. Marks: 70

Note: Answer all the following questions from Section-A and Section-B

Section – A (5x4=20)

Answer all the following questions in not more than **ONE** page each:

1. Prove that the infinits series $\sum_{n=1}^{\infty} \frac{a}{p_n}$ diverges.
2. For n ≥ 1 prove that $\log n = \sum_{d/n} \Lambda(d)$.
3. State and prove Converse of Wilson's theorem.
4. For every odd prime p prove that $\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}} = \begin{cases} 1, & \text{if } p\equiv 1 (\text{modes}) \\ -1, & \text{if } p\equiv 3 (\text{modes}) \end{cases}$
5. Show that 888 is a quadratic non residue of 1999.

Section – B (5x10=50)

Answer all the following questions in not more than **FOUR** pages each:

6 a) State and prove fundamental theorem of arithmetic.

(OR)

b) State and prove the Euclidean algorithm.

7 a) If f is an arithmetical function with $f(1) \neq 0$ then prove that there is a unique arithmetical function f^{-1} called the Dirichlet inverse of f such that $f * f^{-1} = f^{-1} * f = I$ and f^{-1} is given by $f^{-1} = \frac{1}{f(1)}, f^{-1}(n) = \frac{-1}{f(1)} \sum_{\substack{d/n \\ d<n}} f(\frac{n}{d})\, f^{-1}(d)$ for $n > 1$.

(OR)

b) i) If f is multiplicative then prove that $\sum_{d/n} \mu(d)\, f(d) = \prod_{p/n}(1 - f(p))$

ii) For n ≥ 1 prove that $\sigma_{\alpha}^{-1}(n) = \sum_{\frac{d}{n}} d^{\alpha}\, \mu(d)\, \mu(\frac{n}{d})$

8 a) i) If $f(x)=C_0 + C_1 x + \cdots + C_n x^n$ is a polynomial of degree n with integer co-efficients and if $f(x) \equiv 0 (\text{mod } p)$ has more than n solutions where p is a prime then prove that every co-efficient of f(x) is divisible by p.

(OR)

b) State and prove Chinese remainder theorem.

9 a) i) For every odd prime p prove that $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$

ii) Determine whether 219 is a quadratic residue or non residue mod 383.

(OR)

b) State and prove Gauss lemma.

10 a) i) Prove that $a \equiv b \ (\text{mod } m)$ if and only if a and b give the same remainder when divided by n.

ii) Solve the congruence $25x \equiv 15 \ (\text{mod } 120)$

(OR)

b) State and prove quadratic reciprocity law.

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Code No. 1814

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2017

ELEMENTARY NUMBER THEORY

PAPER – IV

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Define divisibility and prove n/n (Reflective).
2. Define Euler-Totient function and find $\emptyset(1), \emptyset(2), \emptyset(3), \emptyset(4)$ values.
3. Prove that congruence is an equivalence relation.
4. Define Legendre symbol.
5. Define multiplicative function.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Let a, b, d, m, n are denote the arbitrary integers then prove that:
   i) $d/n \text{ and } n/m \Rightarrow d/m \ (transitive)$
   ii) $d/n \text{ and } n/m \Rightarrow d/am + bm \ (linear\ property)$
   iii) $d/n \Rightarrow ad/an \ (where\ a \neq 0)$
   iv) $ad/an \text{ and } a \neq 0, then\ d/n$

   (OR)

   b) Prove that every integer n > 1 is either a prime number or a product of prime numbers.

7. a) State and prove Mobius inverse formula.

   (OR)

   b) If f, g are multiplicative functions, then prove that their Dirichlet multiplication f *g is also multiplicative function.

8. a) State and prove Wieson's theorem.

   (OR)

   b) State and prove Chainese Remainder theorem.

9. a) State and prove Euler's criteria.

   (OR)

   b) Legender's symbol $\left(\frac{n}{p}\right)$ is completely multiplicative function. Prove on 'n'.

10. a) State and prove Little Fermat's theorem.

    (OR)

    b) Find the remainder when $(4444)^{4444}$ divided by 9.

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Code No. 1814

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, FEB-2019

ELEMENTARY NUMBER THEORY

PAPER – IV

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. Prove that $(ac, bc) = |c|(a, b)$.
2. Prove that the Dirichlet Product of arthematic functions is Associative.
3. Solve the congruence $5x \equiv 3 \pmod{24}$.
4. Prove that $\left(\frac{mn}{P}\right) = \left(\frac{m}{P}\right)\left(\frac{n}{P}\right)$.
5. Define the Euler totient function $\varphi(n)$ and find $\varphi(100)$.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) Show that there are infinitely many prime numbers. Also, show that every integer $n > 1$ is either a prime number or a product of prime numbers.

(OR)

b) Use Euclidean Algorithm to compute d=(826, 1890). Hence express d as a linear combination of 826 and 1890.

7. a) Prove that for $n \geq 1$, $\sum_{d/n} \varphi(d) = n$

(OR)

b) State and Prove the Mobius inversion formula.

8. a) For any prime P, prove that all the coefficients of the polynomial $f(x) = (x-1)(x-2) \ldots\ldots\ldots\ldots (x-P+1) - x^{P-1} + 1$ are divisible by P.

(OR)

b) State and Prove Chinese remainder theorem.

9. a) Show that, if P is an odd Prime then $\left(\frac{2}{P}\right) = \begin{cases} 1 & \text{if } P \equiv \pm 1 \pmod{8} \\ -1 & \text{if } P \equiv \pm 3 \pmod{8} \end{cases}$

(OR)

b) State and Prove Quadratic reciprocity law.

10. a) For $n \geq 1$, show that $\log n = \sum_{d/n} \Delta(d)$ where $\Delta(d)$ is the mangoldt function.

(OR)

b) State and prove Euler's Fermat theorem.

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Code No. 1814

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY-2022

ELEMENTARY NUMBER THEORY

PAPER – IV

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A (5x4=20)

Answer the following questions in not more than **ONE** page each:

1. If a/bc and if (a,b)=1, then prove that a/c.
2. If f is multiplicative, then prove that $\sum_{d/n} \mu(d)\, f(d) = \prod_{p/n} (1 - f(p))$
3. Solve the quadratic congruence $x^2 \cong 1 \pmod 8$.
4. Find the quadratic residue mod 11.
5. Solve $3x \cong 5 \pmod 7$.

Section – B (5x10=50)

Answer the following questions in not more than **FOUR** pages each:

6. a) State and prove fundamental theorem of arithmetic.

(OR)

b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{P_n}$ diverges.

7. a) If $n \geq 1$, then prove that $\sum_{d/n} \mu(d) = \left[\frac{1}{n}\right] = \begin{cases} 1 & if\ n = 1 \\ 0 & if\ n > 1 \end{cases}$.

(OR)

b) If $n \geq 1$, then prove that $\sum_{d/n} \phi(d) = n$.

8. a) State and prove Euler's Fermat theorem.

(OR)

b) State and prove Wilson's theorem.

9. a) State and prove Gauss Lemma.

(OR)

b) If p is any odd prime, then prove that $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}} \begin{cases} +1 & if\ p \cong \pm 1 \pmod 8 \\ -1 & if\ p \cong \pm 3 \pmod 8 \end{cases}$

10. a) If both g and $f * g$ are multiplicative, then prove that f is multiplicative function.

(OR)

b) State and prove Chinese remainder theorem.

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Code No. 1815

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2016

MATHEMATICAL METHODS

PAPER – V

Time: 3 Hours] [Max. Marks: 70

Note: Answer all the following questions from Section-A and Section-B

Section – A (5x4=20)

Answer all the following questions in not more than **ONE** page each:

1. Solve $\sqrt{p} + \sqrt{q} = 2x$, to obtain the complete integral.
2. Solve $(D^2 - 6DD^1 + 9D^{1^2})z = 6x + 2y$.
3. Show that $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_2(x)$.
4. Show that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$
5. Write orthogonal property of $H_n(x)$.

Section – B (5x10=50)

Answer all the following questions in not more than **FOUR** pages each:

6 a) Prove that Eigen functions corresponding to different eigen values are orthogonal with respect to some weight function.

(OR)

b) Find the equation of the integral surface of the differential equation $2y(z-3)P + (2y-z)q = y(2x-3)$, which passes through the circle $z = 0, x^2 + y^2 = 2x$.

7 a) Solve the one dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 z}{\partial t^2}$ with conditions z(0,t)=z(1,t)=0

(OR)

b) Solve $(D^2 - 3DD^1 + 2D^{1^2})z = e^{2x-y} + e^{x+y} + \cos(x+2y)$.

8 a) State and prove the Rodrigue's formula for Legendre's equation.

(OR)

b) Solve in series $x^2y'' + 2x^2y' - 2y = 0$.

9 a) Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n H_n(x)}{n!}$

(OR)

b) State and prove the generating function for Leguerre polynomial.

10 a) If a>0 prove that $\int_0^\infty e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}$

(OR)

b) Prove that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n . e^{-x})$.

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Code No. 1815

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, DEC-2017

MATHEMATICAL METHODS

PAPER – V

Time: 3 Hours] [Max. Marks: 70

Note: Answer all the following questions from Section-A and Section-B

Section – A (5x4=20)

Answer all the following questions in not more than **ONE** page each:

1. Define Green's function.
2. Classify the partial differential equations

   i) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$

   ii) $(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$
3. Solve $a(P+q) = z$.
4. Solve $\frac{\partial^2 y}{\partial x^2} + y = 0$ by power series method.
5. Prove that $H_n^1(x) = 2n\, H_{n-1}(x)\ (n \geq 1),\ H_0^1(x) = 0$.

Section – B (5x10=50)

Answer all the following questions in not more than **FOUR** pages each:

6 a) Solve $x^{11} + dx = 0,\ x(0) = x^1(1) = 0$ with Strum-Liouilli's method.

(OR)

b) Explain Charpit's method and hence solve $(P^2 + q^2)z = qz$.

7 a) Reduce the equations
$x(xy-1)\, r - (x^2y^2 - 1)\, s + y(xy-1)t + (x-1)P + yq = 0$
to Canonical form and hence solve it.

(OR)

b) Solve $\left(D^2 - DD^1 + 2\, {D^1}^2\right)z = 2x + 3y + e^{3x+4y}$.

8 a) Solve in series of the equation $2x^2y'' + (x^2 - x)y^1 + y = 0$ by Frobenices Method.

(OR)

b) Prove the following

i) $\frac{d}{d_x}\,[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

ii) $x\,J_n^1(x) = -nJ_n(x) + xJ_{n-1}(x)$.

9 a) State and Prove Rodrigue's formula of Hermite Polynomial.

(OR)

b) Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\,(x^n.e^{-x})$.

10 a) Obtain the general solution of one dimensional neat flow equation by the method of separation of variables.

(OR)

b) Express $f(x) = 5x^4 + 8x^3 + 2x^2 - 7x + 4$ in terms of Legendre's Polynomials.

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Code No. 1815

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR EXAMINATIONS, FEB-2019

MATHEMATICAL METHODS

PAPER – V

Time: 3 Hours] [Max. Marks: 70

Note: Answer all the following questions from Section-A and Section-B

Section – A (5x4=20)

Answer all the following questions in not more than **ONE** page each:

1. Solve: $z(z^2 + xy)(px - qy) = x^4$
2. Solve: $(D^2 + DD' + 2D'^2)\, z = e^{x+y}$
3. Solve: $(D^2 - 3DD' + 2D'^2)z = \cos(x + 2y)$
4. Prove that $J_{-1/2}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}}$ cosx
5. Prove that $H_n'(x) = 2n\, H_{n-1}(x);\ (n \geq 1)$

Section – B (5x10=50)

Answer all the following questions in not more than **FOUR** pages each:

6 a) Find the eigen values and eigen functions of the Sturm-Liouville Problem.
$x^{11} + \lambda x = 0,\ x^1(0) = 0, x^1(L) = 0.$

(OR)

b) Use charpit's method to find the complete integral of zpq=p+q

7 a) i) Solve: $(D^2 - DD' + 2\, D'^2)\, z = x + y$

ii) Solve: $\left(D^2 + D'^2\right)z =$ cosmx cosny

(OR)

b) Use the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4\left(\frac{\partial u}{\partial y}\right)$, if $u(0, y) = 8e^{-3y} + 4e^{-5y}$.

8 a) Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ in powers of x.

(OR)

b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0\ if\ m \neq n.$

9 a) Prove that $\int_0^\infty e^{-x}\ L_n(x)\, L_m(x)dx = \begin{cases} 0; if\ m \neq n \\ 1; if\ m = n \end{cases}$

(OR)

b) Express $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$ interms of Harmite's Polynamials.

10 a) Use Lagrange's methods to solve $(y + z)p + (z + x)q = x + y.$

(OR)

b) Prove that $\frac{d}{dx}\left(x^{-n}J_n(x)\right) = -x^{-n}J_{n+1}(x).$

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Code No. 1815

FACULTY OF SCIENCE

M.Sc. (MATHEMATICS) I – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY-2022

MATHEMATICAL METHODS

PAPER – V

Time: 3 Hours] [Max. Marks: 70

Note: Answer all the following questions from Section-A and Section-B

Section – A (5x4=20)

Answer all the following questions in not more than **ONE** page each:

1. Define Green's function.
2. Classify the partial differential equations

   i) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$

   ii) $(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$
3. Prove that $J_{-1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\, cosx$
4. Prove that $H_{n+1}(x) = 2x\, H_n(x) - 2nH_{n-1}(x);\ (n \geq 1)$
5. Use generating function of $P_n(x)$, prove that $P_n(1) = 1$

Section – B (5x10=50)

Answer all the following questions in not more than **FOUR** pages each:

6. a) Find the eigen values and the corresponding eigen functions of Sturm-Liouville Boundary Value Problem $y'' + \lambda y = 0;\ y(0) = 0; and\ y'(L) = 0$

   (OR)

   b) By using Charpit's method, find the complete integral of $zpq = p + q$.

7. a) Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)\, z = 5\sin(2x + y)$.

   (OR)

   b) Reduce the equation $(n-1)^2\frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form, and find its general solution.

8. a) Solve in series of the equation $2x^2y'' + (x^2 - x)y^1 + y = 0$ by Frobenices Method.

   (OR)

   b) Prove the following

   i) $\frac{d}{d_x}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$

   ii) $x\, J_n^1(x) = -nJ_n(x) + xJ_{n-1}(x)$.

9. a) State and Prove Rodrigue's formula of Hermite Polynomial.

   (OR)

   b) Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(x^n . e^{-x}\right)$.

10. a) Obtain the general solution of one dimensional neat flow equation by the method of separation of variables.

    (OR)

    b) Express $f(x) = 5x^4 + 8x^3 + 2x^2 - 7x + 4$ in terms of Legendre's Polynomials.

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